ESD-TR-65-222

A NOTE ON THE CONCEPT OF UNCERTAINTY AS APPLIED IN PSYCHOLOGICAL RESEARCH

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OCTOBER 1965

DECISION SCIENCES LABORATORY
DEPUTY FOR ENGINEERING & TECHNOLOGY
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

Project 7682, Task 768201

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FOREWORD

This research was performed at the Decision Sciences Laboratory, Electronic Systems Division, AF Systems Command as part of Project 7682, Man-Computer Information Processing, Task 768201, Data Presentation and Human Data Processing.

This Technical Documentary Report has been reviewed and is approved.

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ABSTRACT

This note distinguishes four different connotations of "uncertainty" as the term has been used in the psychological literature.

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As defined by Shannon, the average amount of information (or uncertainty) associated with the occurrence of an event with n possible outcomes is

$$H(x) = -\sum_{i=1}^{n} p(x_i) \log_2 p(x_i) \tag{1}$$

where $p(x_i)$ is the probability of the outcome x_i.

Both the uncertainty measure and the associated conceptual framework have been major determinants of many recent trends in experimental psychology. Several tutorial expositions of theory, and reviews of related empirical studies are available (e.g., Attneave, 1959 ; Luce, 1960). The purpose of this note is to distinguish several possible connotations of H as it has been used in the psychological literature.

1. Perhaps the most straight forward use of uncertainty is as a measure of non-metric variability. In this sense it is a statistical property of a categorized data, and is given by

$$H_s(x) = -\sum_{i=1}^{n} \frac{N(x_i)}{T} \log_2 \frac{N(x_i)}{T} \tag{2}$$

where $N(x_i)$ represents the number of occurrences of the outcome x_i and T is the total number of events in the sample. Uncertainty analysis is similar in some respects to variance analysis and has the advantage that it can be applied to any set of data which is nominal or can be reduced to nominal from (Garner & McGill, 1956). The use of uncertainty as a descriptive statistic requires no assumptions concerning underlying distributions, sampling procedures, or a priori subjective probabilities of subjects.

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2. The term also has been used to connote a <u>parameter of a theoretical probability distribution</u>, which is not actually measured, but is given by definition, or inferred from sample statistics. This usage will be denoted by

$$H_p(x) = -\sum_{i=1}^{n} p_p(x_i) \; log_2 \; p_p(x_i) \tag{3}$$

where $p_p(x_i)$ represents the defined, or inferred, probability of the outcome x_i. The uncertainties associated with a "fair" toss of an "unbiased" coin, and with a roll of a perfect die, are by definition 1 and 2.58 bits respectively. Whether or not there are such things as an unbiased coin or a perfect die is irrelevant. Even if there were, however, we would expect that with a finite sample of tosses or rolls, the statistic $H_s(x)$ would be somewhat less than the parameter $H_p(x)$ since all possible outcomes are unlikely to occur with exactly equal frequency in either case. With small samples of events with several possible outcomes the difference between $H_p(x)$ and $H_s(x)$ can be quite large. Miller (1955) has shown that with samples drawn from a distribution with the parameter $H_p(x)$, in general $H_s(x)$ will be smaller than $H_p(x)$ by an amount proportional to the number of different possible outcomes and inversely proportional to the number of observations in the sample.

3. Frequently in psychological experiments, the stimulus selection procedure actually used by *E* is not strictly consistent with the information given to *S* concerning the probabilities associated with the outcomes which could occur. For example, *S* may be told that each of *k* stimuli is equally

likely to occur on each trial of the experiment, whereas the stimulus selection procedure involves constraints such as the forcing of an equal number of occurrences of each alternative during some segment of an experimental session, or the avoidance of runs exceeding some predetermined length. We will represent average uncertainty, as implied by a selection procedure, as

$$H_c(x) = -\frac{1}{n} \sum_{i=1}^{m} p_n\{s_i\} \log_2 p_n\{s_i\} \tag{4}$$

where $p_n\{s_i\}$ is the probability of the occurrence of the n-tuple sequence $\{s_i\}$ in a sample of size n, and m is the total number of different sequences possible given the sampling constraints. This formula simply makes use of the fact that one may calculate the average information in an event by calculating the average information in a sequence of events and dividing by the number of events in the sequence. In the case of no sampling constraints, i.e., independent sampling on each trial, (3) and (4) are equivalent, but (3) is easier to compute. However, when forcing constraints are employed, (3) is inappropriate since $p(x_i)$ changes from trial to trial as a function of what events have already been selected. As an example of the possible outcomes of forcing constraints on H, consider the experiment, four successive tosses of an unbiased coin. Since there are 16 possible sequences of heads and tails, each with probability 1/16, the uncertainty associated with the outcome of the experiment is four bits, or one bit per toss. However, if the experiment were constrained to insure that the total number of heads would equal the total number of tails, then there would be only six possible outcomes, and the experiment would now be

worth not more than 2.6 bits, or an average uncertainty of somewhat less than .7 bits per toss. Note, however, that the statistic $H_s(x)$ when calculated on the data from such an experiment would be insensitive to the forcing constraint and would yield an average uncertainty of one bit per toss.

In general, $H_s(x)$, $H_p(x)$ and $H_c(x)$ will not be equal in any particular case. More specifically, if $H_s(x)$ and $H_p(x)$ are equal, $H_c(x)$ will be different; if $H_p(x)$ and $H_c(x)$ are equal, $H_s(x)$ will be different. For example, consider an experiment in which *S* is told that each of eight stimuli is equally likely to occur on each of 64 trials, when, in fact, the sampling procedure is constrained so as to force exactly two occurrences of each stimulus in each successive block of 16 trials. In this case, the instructions to the subject imply that $H_p(x)$ is three bits, and the appropriate calculation would show that $H_s(x)$ also is three bits; however, because of the sampling constraint, $H_c(x)$ is less than 3 bits. If on the other hand the selection procedure is consistent with the instructions, and the selection of the stimulus on each trial is independent of all previous selections, then $H_p(x)$ and $H_c(x)$ both equal three bits, but in general, with such a sampling procedure, all x_i will not occur with equal frequency, hence, $H_s(x)$ will be less than three bits. One of the purposes of the use of sampling constraints is, of course, to force $H_s(x)$, as computed from a stimulus sample, to correspond exactly to $H_p(x)$ as implied by the outcome probabilities stated to *S*. When this is the case, in general $H_c(x) \neq H_p(x)$.

4. A fourth use of the concept has been to connote <u>an observer's or receiver's uncertainty</u> with respect to which of a set of possible outcomes or

message elements will occur. One's degree of uncertainty with respect to a particular outcome is intuitively analogous to the degree to which he would be surprised by its occurrence - the inverse of the degree to which he expects it to occur. An outcome which is expected causes little surprise and gives little information by its occurrence; whereas the occurrence of an outcome which was considered unlikely is both surprising and, at least in a technical sense, informative.

In this case we say

$$H_e(x) = -\sum_{i=1}^{n} p_e(x_i) \log_2 p_e(x_i) \tag{5}$$

where $p_e(x_i)$ represents the relative likelihood or probability which the receiver associates with the occurrence of the outcome x_i. Whereas one might assume the expectancies of an "ideal" receiver to be consistent with the available relevant information concerning the set of alternatives and the sampling rules, $p_e(x)$ represents the expectancies of a human receiver and may be biased by irrelevancies, or by unfounded assumptions about probabilistic events that he brings to the situation, e.g., the so-called "gambler's fallacy" of assuming sequential dependencies in a series of independent events. (We should note that in view of the forcing constraints that experimenters frequently impose on randomization procedures, the gambler's fallacy often is not so fallacious in the experimental situation as has been supposed.)

Although $H_e(x)$ is the measure which is most directly relevant to questions of human information processing capabilities, it is by far the most difficult to assess. Certainly the assumption that $H_e(x)$ corresponds exactly to either $H_s(x)$, $H_p(x)$ or $H_c(x)$ is not warranted in general. That

this is so is intuitively clear from the fact that different receivers may gain different amounts of information - may be surprised to different degrees - by the occurrence of the same message or outcome. Cronbach (1955) has shown that a receiver may gain more information from a message if his a priori expectancies are in error than if they in fact are consistent with the properties of the source. Moreover, one's ability to state outcome probabilities, or to describe the process by which *E* selects the outcomes, does not reveal the nature of his expectancies on individual trials of an experiment. It seems likely that even in the case of well informed and mathematically sophisticated individuals expectancies may be subject to trial by trial variations resulting from idiosyncratic guessing strategies, memory limitations, and momentary attention shifts.

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Unclassified

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Decision Sciences Laboratory L. G. Hanscom Field, Bedford, Mass	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP n/a

3. REPORT TITLE

A Note on The Concept of Uncertainty As Applied in Psychological Research

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

5. AUTHOR(S) *(Last name, first name, initial)*

Nickerson, Raymond S.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
October 1965	11	5

8a. CONTRACT OR GRANT NO. In-House b. PROJECT NO. 7682 c. Task 768201 d.	9a. ORIGINATOR'S REPORT NUMBER(S) ESD-TR-65-222 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. AVAILABILITY/LIMITATION NOTICES

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
None	Electronic Systems Division, Air Force Systems Command, USAF, L. G. Hanscom Field, Bedford, Massachusetts 01731

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DD FORM 1 JAN 64 1473

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Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Information Theory Psychology						

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